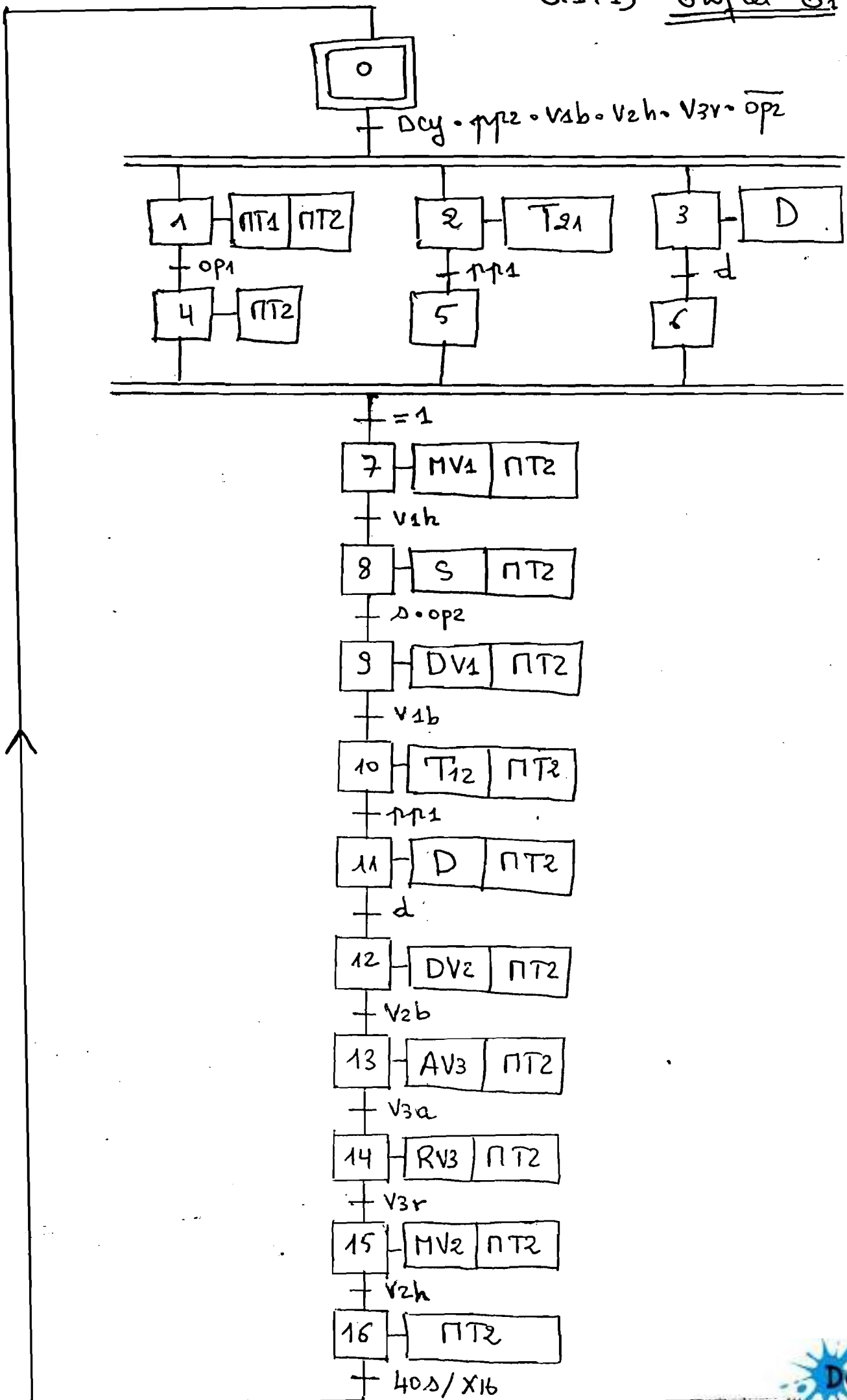
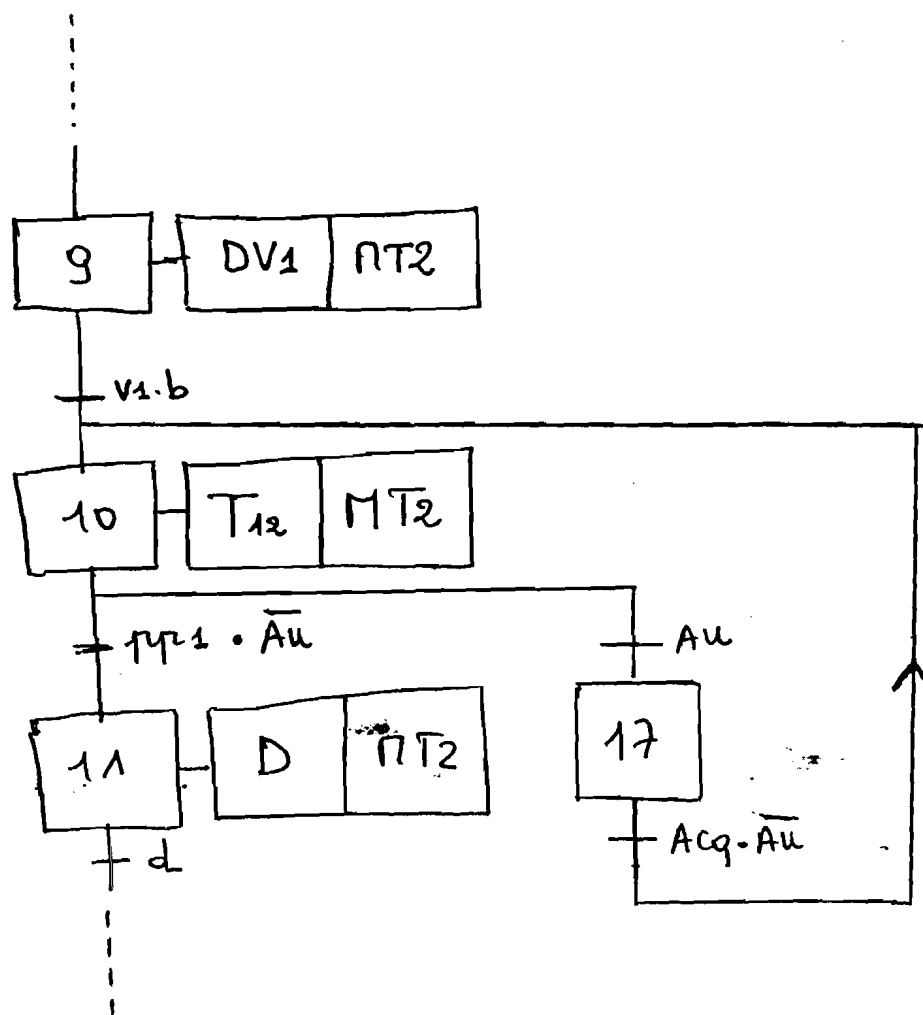




C.I.2)

Grafset G₁ modifié



Partie C-II - Asservissement de position d'un axe du bras robotisé

II-1 : Simplification du schéma fonctionnel de la figure 6 :

$$\Omega_m(p) = \frac{1}{J_m p + f_m} \left[\frac{k_c}{R} (U(p) - k_e \Omega_m(p)) - \frac{1}{e} \left(C_r(p) + \frac{1}{e} (J_c p + f_c) \Omega_m(p) \right) \right]$$

$\Leftrightarrow$

$$\left[1 + \frac{k_e k_c / R}{J_m p + f_m} + \frac{1}{e^2} \frac{J_c p + f_c}{J_m p + f_m} \right] \Omega_m(p) = \frac{k_c / R}{J_m p + f_m} U(p) - \frac{1/e}{J_m p + f_m} C_r(p)$$

$\Leftrightarrow$

$$\left[\left(J_m + \frac{J_c}{e^2} \right) p + \left(f_m + \frac{f_c}{e^2} \right) + \frac{k_e k_c}{R} \right] \Omega_m(p) = \frac{k_c}{R} U(p) - \frac{1}{e} C_r(p)$$

$\Rightarrow$

$$\Omega_m(p) = T_1(p) U(p) - T_2(p) C_r(p)$$

avec

$$T_1(p) = \frac{K_1}{1 + \tau_{em} p} \quad \text{et} \quad T_2(p) = \frac{K_2}{1 + \tau_{em} p}$$

$$\tau_{em} = \frac{R J_e}{k_e k_c + f_e R} : \text{constante de temps électromécanique}$$

$$K_1 = \frac{k_c}{k_e k_c + R f_e} : \text{Gain statique de } T_1(p)$$

$$K_2 = \frac{R}{e (k_e k_c + R f_e)} : \text{Gain statique de } T_2(p)$$

$$J_e = J_m + \frac{J_c}{e^2} : \text{Inertie équivalente ramenée sur l'axe du moteur}$$

$$f_e = f_m + \frac{f_c}{e^2} : \text{Coef. de frottement visqueux équivalent ramenée sur l'axe du moteur}$$

II-2: A partir de la réponse indicielle de $T_1(p)$ à un échelon de tension d'amplitude 25V:

On a:

$$\omega_m(\infty) = 25 K_1 = 200 \text{ rad.s}^{-1} \Rightarrow K_1 = 8 \text{ rad.s}^{-1} \text{ V}^{-1}$$

à 63% de $\omega_m(\infty)$ on trouve $\tau_{em} = 10 \text{ ms}$

II-3: schéma fonctionnel de la figure 8.

a) Fonction de transfert en boucle fermée:

$$\Theta_c(p) = H_1(p) \Theta_{ref}(p) - H_2(p) C_r(p)$$

$$\text{avec } H_1(p) = \left. \frac{\Theta_c(p)}{\Theta_{ref}(p)} \right|_{C_r=0} = \frac{\frac{\alpha A K_1}{e \tau_{em}}}{p^2 + \frac{1}{\tau_{em}} p + \frac{\alpha A K_1}{e \tau_{em}}}$$

$$H_2(p) = \left. \frac{\Theta_c(p)}{C_r(p)} \right|_{\Theta_{ref}=0} = \frac{\frac{K_R}{e \tau_{em}}}{p^2 + \frac{1}{\tau_{em}} p + \frac{\alpha A K_1}{e \tau_{em}}}$$

Equation caractéristique:

$$p^2 + \frac{1}{\tau_{em}} p + \frac{\alpha A K_1}{e \tau_{em}} = p^2 + 2m\omega_0 p + \omega_0^2$$

Par identification, on obtient:

$$\omega_0 = \sqrt{\frac{\alpha A K_1}{e \tau_{em}}} : \text{ pulsation propre non amortie (rad/s)}$$

$$m = \frac{1}{2} \sqrt{\frac{e}{\alpha A K_1 \tau_{em}}} : \text{ coefficient d'amortissement.}$$

b- Calcul de A pour avoir $m=0,7$.

$$\alpha = 0,8 \text{ V/rad} ; K_1 = 8 \text{ rad.s}^{-1} \text{ V}^{-1} ; \tau_{em} = 10 \text{ ms et } e = 50$$

$$A = \frac{e}{L m^2 \alpha K_1 \tau_{em}}$$

$$\text{A.N: } A = \frac{50}{4 \cdot (0,7)^2 \cdot 0,8 \cdot 8 \cdot 0,01} = 398,6$$

$$\omega_0 = \sqrt{\frac{0,8 \cdot 398,6 \cdot 8}{50 \cdot 0,01}} = 71,43 \text{ rad/s}$$

$$\text{Déphasement: } D\% = 100 \cdot e^{-\frac{m\pi}{\sqrt{1-m^2}}}$$

$$\text{A.N: } \boxed{D\% = 4,6\%}$$

$$\text{Temps de pic: } T_p = \frac{\pi}{\omega_0 \sqrt{1-m^2}}$$

$$\text{A.N: } \boxed{T_p = 0,062 \text{ s}}$$

c- Etude de la précision statique

$$\boxed{E_1(p) = \Theta_{ref}(p) - \Theta_c(p) = [1 - H_1(p)] \Theta_{ref}(p) + H_2(p) C_r(p)}$$

$$\text{avec } H_1(p) = \frac{\omega_0^2}{p^2 + 2m\omega_0 p + \omega_0^2}$$

$$H_2(p) = \frac{K_2}{\alpha A K_1} \cdot \frac{\omega_0^2}{p^2 + 2m\omega_0 p + \omega_0^2} = \frac{K_2}{\alpha A K_1} H_1(p)$$

$$\boxed{E_1(p) = \frac{p(p + 2m\omega_0)}{p^2 + 2m\omega_0 p + \omega_0^2} \Theta_{ref}(p) + \frac{K_2}{\alpha A K_1} \frac{\omega_0^2}{p^2 + 2m\omega_0 p + \omega_0^2} C_r(p)}$$

$$\begin{aligned} \theta_{ref}(t) = t \cdot u(t) &\xrightarrow{T.L.} \theta_{ref}(p) = \frac{1}{p^2} \\ C_r(t) = 100 \cdot u(t) &\xrightarrow{T.L.} C_r(p) = \frac{100}{p} \end{aligned}$$

Théorème de la valeur finale

$$E_1(\infty) = \lim_{p \rightarrow 0} p E_1(p)$$

soit

$$E_1(\infty) = \frac{2m}{w_0} + \frac{100 K_2}{\alpha A K_1}$$

A.N: $E_1(\infty) = 0,0196 + 0,026 = 0,049$

$$E_1(\infty) = 0,049 \text{ soit } 4,9\%$$

Partie A: Géométrie des masses.

(I)

I. 1. Position du centre G de l'ensemble (4)

- cylindre (masse M)

$$\vec{KG}_c = \frac{H}{2} \vec{x}_3 + L \vec{y}_1$$

- tige (masse m_t)

$$\vec{KG}_t = \left(\frac{L}{2} - a_4\right) \vec{y}_1$$

- disque (masse m)

$$\vec{KG}_d = -a_4 \vec{y}_1$$

On applique la relation du barycentre

$$\vec{KG} = \frac{M \vec{KG}_c + m_t \vec{KG}_t + m \vec{KG}_d}{M + m_t + m}$$

$$x = \frac{M \frac{H}{2}}{M + m_t + m}$$

$$y = \frac{ML + m_t \left(\frac{L}{2} - a_4\right) - m a_4}{M + m_t + m}$$

I. 2. Matrices centrales d'inertie

- cylindre:

$$[I_{(G)}] = \begin{bmatrix} \frac{Mr^2}{2} & 0 & 0 \\ 0 & \frac{Mr^2}{4} + \frac{MH^2}{12} & 0 \\ 0 & 0 & \frac{Mr^2}{4} + \frac{MH^2}{12} \end{bmatrix}$$

($\vec{x}_3, \vec{y}_1, \vec{z}_1$)

$$A_4 = \frac{Mr^2}{2} + ML^2 + \frac{m_t L^2}{12} + m_t \left(\frac{L}{2} - a_4\right)^2 + \frac{mr^2}{4} + m a_4^2$$

$$B_4 = \frac{Mr^2}{4} + \frac{MH^2}{12} + \frac{mr^2}{2} + \frac{MH^2}{4}$$

$$C_4 = \frac{Mr^2}{4} + \frac{MH^2}{12} + \frac{MH^2}{4} + ML^2 + \frac{m_t L^2}{12} + \left(\frac{L}{2} - a_4\right)^2 m_t + \frac{mr^2}{4} + m a_4^2$$

$$F_4 = \frac{MH}{2} L$$

Partie A. II. Cinématique.

II. 1 Vitesses

$$\vec{v}_{M/R_0} = \vec{v}_{O/R_0} + \vec{\omega}_{1/R_0} \wedge \vec{OA}$$

$$\vec{v}_{O/R_0} = \vec{0}, \quad \vec{\omega}_{1/R_0} = \dot{\beta} \vec{x}_3, \quad \vec{OA} = a_4 \vec{y}_1$$

$$\vec{v}_{M/R_0} = a_4 \dot{\beta} \vec{z}_1$$

$$\vec{v}_{I/R_0} = \vec{v}_{O/R_0} + \vec{\omega}_{1/R_0} \wedge \vec{OI}$$

$$\vec{OI} = a_3 \vec{x}_3 + b_4 \vec{y}_1$$

$$\vec{v}_{I/R_0} = b_4 \dot{\beta} \vec{z}_1$$

$$[\mathbb{I}(\text{CCY})] = \begin{bmatrix} 0 & \frac{Mr^2}{4} + \frac{MH^2}{12} & 0 \\ 0 & 0 & \frac{Mr^2}{4} + \frac{MH^2}{12} \end{bmatrix} (\vec{x}_3, \vec{y}_1, \vec{z}_3)$$

• tige.

$$[\mathbb{I}(\text{tige})] = \begin{bmatrix} \frac{m_k L^2}{12} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{m_k L^2}{12} \end{bmatrix} (\vec{x}_3, \vec{y}_1, \vec{z}_3)$$

• disque.

$$[\mathbb{I}(\text{disque})] = \begin{bmatrix} \frac{mr^2}{4} & 0 & 0 \\ 0 & \frac{mr^2}{2} & 0 \\ 0 & 0 & \frac{mr^2}{4} \end{bmatrix} (\vec{x}_3, \vec{y}_1, \vec{z}_3)$$

I.3 Matrice d'inertie de l'ensemble (4).

le plan $(K, \vec{x}_3, \vec{y}_1)$ est un plan de symétrie,
par suite $(K, \vec{z}_3)$ est un axe principal d'inertie

$$[\mathbb{I}_K(u)] = \begin{bmatrix} A_u & -F_u & 0 \\ -F_u & B_u & 0 \\ 0 & 0 & C_u \end{bmatrix} (\vec{x}_3, \vec{y}_1, \vec{z}_3)$$

avec:

$$\vec{V}_{I/R_0} = b_1 \dot{\beta} \vec{z}_1$$

$$\vec{V}_{K/R_0} = \vec{V}_{O/R_0} + \vec{\Omega}_{1/R_0} \wedge \vec{OK}$$

$$\vec{OK} = a_3 \vec{x}_0 + b_2 \vec{y}_1$$

$$\vec{V}_{K/R_0} = b_2 \dot{\beta} \vec{z}_1$$

$$\vec{V}_{EE2/R_1} = \vec{V}_{K/R_1} + \vec{\Omega}_{2/R_1} \wedge \vec{AE}$$

$$\vec{V}_{A/R_1} = \vec{0}, \quad \vec{\Omega}_{2/R_1} = \dot{\theta} \vec{x}_0, \quad \vec{AE} = c_1 \vec{x}_0 + r_2 \vec{y}_1$$

$$\vec{V}_{EE2/R_1} = r_2 \dot{\theta} \vec{z}_1$$

$$\vec{V}_{EE3/R_1} = \vec{V}_{I/R_1} + \vec{\Omega}_{3/R_1} \wedge \vec{IE}$$

$$\vec{V}_{I/R_1} = \vec{0}, \quad \vec{\Omega}_{3/R_1} = \dot{\psi} \vec{y}_1, \quad \vec{IE} = -r_3 \vec{x}_0 - c_2 \vec{y}_1$$

$$\vec{V}_{EE3/R_1} = r_3 \dot{\psi} \vec{z}_1$$

$$\vec{V}_{DE5/R_2} = \vec{V}_{A/R_2} + \vec{\Omega}_{4/R_2} \wedge \vec{AD}$$

$$\vec{V}_{A/R_2} = a_1 \dot{\beta} \vec{z}_1, \quad \vec{AD} = a_2 \vec{x}_0 - r_5 \vec{y}_1, \quad \vec{\Omega}_{4/R_2} = (\dot{\alpha} + \dot{\theta}) \vec{z}_1$$

$$\vec{v}_{D \in \mathcal{C}/R_0} = a_1 \dot{\beta} \vec{z}_1 - r_f (\dot{\beta} + \dot{\theta}) \vec{z}_1 \quad (\text{Page 2})$$

II. 2 Roulement sans glissement aux points D et E

• en E

$$\vec{v}_{E \in 2/R_0} = \vec{v}_{E \in 3/R_0}$$

$$r_2 \dot{\theta} = r_3 \dot{\psi} \Rightarrow$$

$$\dot{\psi} = \frac{r_2}{r_3} \dot{\theta} = \frac{r_2}{r_3} \omega$$

• En D : $\vec{v}_{D \in 6/R_0} = \vec{0}$

$$a_1 \dot{\beta} - r_f (\dot{\beta} + \dot{\theta}) = 0$$

$$(a_1 - r_f) \dot{\beta} = r_f \dot{\theta}$$

$$\dot{\beta} = \frac{r_f}{a_1 - r_f} \dot{\theta} = \frac{r_f}{a_1 - r_f} \omega$$

I. 3 Vitesse de G.

$$\vec{v}_{G/R_0} = \vec{v}_{K/R_0} + \vec{\omega}_{3/R_0} \wedge \vec{KG}$$

$$\vec{v}_{K/R_0} = b \dot{\beta} \vec{z}_1, \quad \vec{\omega}_{3/R_0} = \dot{\psi} \vec{y}_1 + \dot{\beta} \vec{x}_0$$

$$\vec{KG} = z \vec{x}_3 + y \vec{y}_1$$

$$\vec{v}_{G/R_0} = b \dot{\beta} \vec{z}_1 + (\dot{\psi} \vec{y}_1 + \dot{\beta} \vec{x}_0) \wedge (z \vec{x}_3 + y \vec{y}_1)$$

$$M_x = -x \dot{\psi}^2 \cos \varphi$$

(I)

$$M_y = x \dot{\beta} \dot{\psi} \cos \varphi - \dot{\beta} (b_u + y) \dot{\beta} - x \dot{\psi} \cos \varphi$$

$$M_z = x \dot{\psi}^2 \sin \varphi + z \dot{\beta}^2 \sin \varphi$$

Partie A. III. Dynamique

$$\varphi = 0, \quad \dot{\varphi} \neq 0$$

$$\vec{z}_3 = \vec{z}_1, \quad \vec{y}_3 = \vec{y}_1, \quad \vec{x}_3 = \vec{x}_1$$

III. 1 Action sur (4)

• Poids appliqué en G. $\vec{P} = -(\overbrace{M+m_t+m}^{m_4})g \vec{z}_0$

$$\vec{M}_G(\vec{P}) = \vec{0}, \quad \vec{P} = -m_4 g (\cos \beta \vec{z}_1 + \sin \beta \vec{y}_1)$$

$$\vec{M}_K(\vec{P}) = (x \vec{x}_0 + y \vec{y}_1) \wedge (m_4 g$$

$$= -m_4 g \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ \sin \beta \\ \cos \beta \end{pmatrix}$$

$$= -m_4 g \begin{pmatrix} y \cos \beta \\ x \cos \beta \\ x \sin \beta \end{pmatrix}$$

$$KG = z\vec{x}_3 + y\vec{y}_1$$

$$\vec{v}_{G/R_0} = b\dot{\beta}\vec{z}_1 + (\dot{\psi}\vec{y}_1 + \dot{\beta}\vec{x}_0) \wedge (z\vec{x}_3 + y\vec{y}_1)$$

$$\vec{v}_{G/R_0} = b\dot{\beta}\vec{z}_1 - x\dot{\psi}\vec{z}_3 + x\dot{\beta}\sin\psi\vec{y}_1 + y\dot{\beta}\vec{z}_1$$

$$\vec{v}_{G/R_0} = (b+y)\dot{\beta}\vec{z}_1 - x\dot{\psi}\vec{z}_3 + x\dot{\beta}\sin\psi\vec{y}_1$$

$$\vec{z}_3 = \cos\psi\vec{z}_1 + \sin\psi\vec{x}_0$$

$$\vec{v}_{G/R_0} = -x\dot{\psi}\sin\psi\vec{x}_0 + x\dot{\beta}\sin\psi\vec{y}_1 + ((b+y)\dot{\beta} - x\dot{\psi}\cos\psi)\vec{z}_1$$

$$v_x = -x\dot{\psi}\sin\psi, \quad v_y = x\dot{\beta}\sin\psi$$

$$v_z = (b+y)\dot{\beta} - x\dot{\psi}\cos\psi$$

I.4. Accélération de G.

$$\vec{a}_{G/R_0} = \frac{d\vec{v}_{G/R_0}}{dt} \Big|_R$$

$$\vec{a}_{G/R_0} = \dot{v}_x\vec{x}_0 + \dot{v}_y\vec{y}_1 + \dot{v}_z\vec{z}_1 + \dot{\beta}v_y\vec{z}_1 - v_z\dot{\beta}\vec{y}_1$$

avec $\dot{v}_x = -x\dot{\psi}^2\cos\psi$, $\dot{v}_y = x\dot{\beta}\dot{\psi}\cos\psi$,
 $\dot{v}_z = x\dot{\psi}^2\sin\psi$.

$$= -m_4g \begin{pmatrix} x\cos\beta \\ x\sin\beta \end{pmatrix}$$

$$\{\mathcal{E}(p)\}_K = \left\{ \begin{array}{c|c} 0 & -m_4gy\cos\beta \\ -m_4g\sin\beta & m_4gx\cos\beta \\ -m_4g\cos\beta & -m_4gx\sin\beta \end{array} \right\}_{(\vec{x}_0, \vec{y}_1, \vec{z}_1)}$$

• Action de 3 sur (u) en K

$$\{\mathcal{E}_{4/3}\}_K = \left\{ \begin{array}{c|c} x_{34} & L_{14} \\ y_{34} & 0 \\ z_{34} & N_{14} \end{array} \right\}_{(x_0, y_1, z_1)}$$

• action de (3) sur (u) en K.

$$\{\mathcal{E}_{4/3}\}_K = \left\{ \begin{array}{c|c} 0 & 0 \\ N_J & G_J \\ 0 & 0 \end{array} \right\}_{(\vec{x}_0, \vec{y}_1, \vec{z}_1)}$$

III. 3 Théorème de la résultante dynamique

$$\begin{cases} x_4 = -m_4x\dot{\psi}^2 \\ N_J + Y_{14} - m_4g\sin\beta = (2x\dot{\beta}\dot{\psi} - \dot{\beta}^2(b+y))m_4 \\ z_4 - m_4g\cos\beta = 0 \end{cases}$$

(Page 3)

II.3) Moment dynamique en K.

$$\vec{S}_K(4/R_0) = m_4 \vec{KG} \wedge \vec{V}_{K/R_0} + \left[\mathbb{I}_K^{(4)} \right] \vec{\omega}_{4/R_0}$$

$$\vec{S}_K(4/R_0) = \frac{d\vec{S}_K(4/R_0)}{dt} + m_4 \vec{V}_{K/R_0} \wedge \vec{V}_{G/R_0}$$

$$\begin{aligned} \vec{KG} \wedge \vec{V}_{K/R_0} &= (x \vec{x}_3 + y \vec{y}_1) \wedge (b_2 \dot{\beta} \vec{z}_1) \\ &= -b_2 \dot{\beta} x \cos \varphi \vec{y}_1 + b_2 \dot{\beta} y \vec{x}_3 \end{aligned}$$

$$\vec{S}_{4/R_0} = \dot{\varphi} \vec{y}_1 + \dot{\theta} (\cos \varphi \vec{x}_3 + \sin \varphi \vec{z}_3)$$

$$\left[\mathbb{I}_K^{(4)} \right] \cdot \vec{S}_{4/R_0} = \begin{bmatrix} A_4 & -F_4 & 0 \\ -F_4 & B_4 & 0 \\ 0 & 0 & C_4 \end{bmatrix} \begin{bmatrix} \dot{\theta} \cos \varphi \\ \dot{\varphi} \\ \dot{\theta} \sin \varphi \end{bmatrix}$$

$$= \begin{bmatrix} A_4 \dot{\theta} \cos \varphi - F_4 \dot{\varphi} \\ -F_4 \dot{\theta} \cos \varphi + B_4 \dot{\varphi} \\ C_4 \dot{\theta} \sin \varphi \end{bmatrix}$$

$$\vec{S}_K(4/R_0) = \left(B_4 \dot{\beta} \dot{\varphi} - F_4 \dot{\theta} \dot{\beta} - m_4 b_2 \dot{\beta}^2 x + (C_4 - A_4) \dot{\theta} \dot{\varphi} + F_4 \dot{\varphi}^2 \right) \vec{z}_1$$

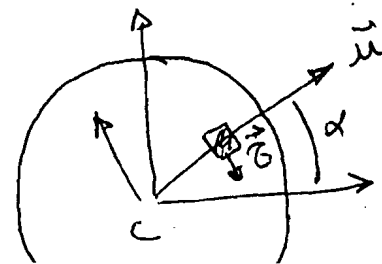
Equations dynamiques:

$$\begin{cases} A_4 - m_4 g y \cos \beta = 0 \\ C_4 + m_4 g x \cos \beta = 0 \\ N_{14} = (B_4 \dot{\beta} \dot{\varphi} + F_4 (\dot{\varphi}^2 - \dot{\theta} \dot{\beta})) - m_4 b_2 \dot{\beta}^2 + (C_4 - A_4) \dot{\theta} \dot{\varphi} \end{cases}$$

III.4 Expression de N_J

On considère une surface d'appui circulaire de rayon R .
la pression est uniforme:

$$p = \frac{N_J}{S} = \frac{N_J}{\pi R^2}$$



$$\vec{\sigma}_4(u/R_0) = -m_4 b_2 \dot{\beta} x \omega \varphi \vec{Y}_1 + m_4 b_2 \dot{\beta} y \vec{X}_0 +$$

$$(A_4 \ddot{\theta} \omega \varphi - F_4 \dot{\varphi}) \vec{X}_3 +$$

$$(-F_4 \ddot{\theta} \omega \varphi + B_4 \dot{\varphi}) \vec{Y}_1 +$$

$$C_4 \ddot{\theta} \sin \varphi \vec{Z}_3$$

$$\frac{d\vec{\sigma}_K}{dt} \Big|_R = (m_4 b_2 \dot{\beta} \dot{\varphi} x \sin \varphi) \vec{Y}_1 +$$

$$\ddot{\theta} (B_4 \dot{\varphi} - F_4 \ddot{\theta} \omega \varphi - m_4 b_2 \dot{\beta} x \omega \varphi) \vec{Z}_1$$

$$- A_4 \ddot{\theta} \dot{\varphi} \sin \varphi \vec{X}_3 + (A_4 \ddot{\theta} \omega \varphi - F_4 \dot{\varphi}) (\ddot{\theta} \sin \varphi \vec{Y}_1 - \dot{\varphi} \vec{Z}_3)$$

$$+ C_4 \ddot{\theta} \dot{\varphi} \omega \varphi \vec{Z}_3 + C_4 \ddot{\theta} \sin \varphi (\dot{\varphi} \vec{X}_3 - \ddot{\theta} \omega \varphi \vec{Y}_1)$$

$$\frac{d\vec{\sigma}_K}{dt} \Big|_R = \ddot{\theta} (B_4 \dot{\varphi} - F_4 \ddot{\theta} - m_4 b_2 \dot{\beta} x) \vec{Z}_1 +$$

$$- (A_4 \ddot{\theta} - F_4 \dot{\varphi}) \dot{\varphi} \vec{Z}_1 + C_4 \ddot{\theta} \dot{\varphi} \vec{Z}_1$$

$$m_4 \vec{V}_{K/R_0} \wedge \vec{V}_{G/R_0} = m_4 \cdot b_2 \dot{\beta} \vec{Z}_1 \wedge ((b_2 + y) \dot{\beta} - x \dot{\varphi}) \vec{Z}_1$$

$$= \vec{0}$$

$$\vec{\sigma} = -\tau \vec{v} = -f.p. \vec{v}$$

à la limite de l'adhérence $\frac{|\vec{\sigma}|}{\rho} = f$.

$$d\vec{F} = -\tau ds \vec{v} = -\tau r dr d\alpha \vec{v}$$

$$d\vec{M}_C = \vec{CH} \wedge d\vec{F}$$

$$= -r \vec{u}_n \tau r dr d\alpha \vec{v}$$

$$= -\tau r^2 dr d\alpha \vec{z}$$

$$\vec{M}_J = \int \tau r^2 dr d\alpha = \frac{2\pi \tau \cdot R^3}{3}$$

$$C_J = - \frac{2\pi \tau \cdot R^3}{3} = -\frac{2\pi N_J R}{3}$$

$$N_J = |G| \cdot \frac{3}{2fR}$$

III.5 inconnues de liaisons.

$$N_J = 3 \cdot \frac{m_4 g x |\cos \beta|}{2fR}$$

$$L_{14} = m_4 g y \cos \beta$$

$$Y_{14} = m_4 g \sin \beta + m_4 (2x \dot{\beta} \dot{\varphi} - \dot{\beta}^2 (b_2 + y)) - 3m_4 x \ddot{\beta}$$

$$Z_{14} = m_4 g \cos \beta$$